

GRADUATE SEMINAR ON LOGIC (S4A4) SOSE 2024

O-minimal structures

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Time and Place. Fridays 12.15 -14, Endenicher Allee 60, SemR 0.011.
The seminar will not meet on Fridays after a public holiday.

Organizational meeting. The organizational meeting will take place on Tuesday January 23rd at 10.15am (s.t.) in N0.008 (Neubau Endenicher Allee 60).

If you can't make that meeting (or missed the meeting) but want to give a talk, please send an email to the instructor (by February 1st), indicating at least three topics you are willing give a talk on and whether you are planning to V4A7 Adv Math Logic I next semester (or have taken a similar course).

Abstract. Originated in logic, o-minimality has recently seen many applications in number theory and geometry. This seminar serves as an introduction to the construction of o-minimal structures on the real field. The main results discussed in this seminar will be the o-minimality of the following structures:

- \mathbb{R}_{an} , the real field with all restricted analytic function ([1]),
- \mathbb{R}_{exp} , the real field with the exponential function ([6]),
- $(\mathbb{R}_{\text{an}}, \text{exp})$, the real field with the exponential function and all restricted analytic functions ([5, 4]),
- $(\mathbb{R}_{\text{Pfaff}})$, the real field expanded by all Pfaffian functions ([7]).

While the consequences of o-minimality are outlined in [3] (and will be covered in the lecture V4A7 Advanced Mathematical Logic I), the book does not contain much about how to prove o-minimality of a given structure and postpones this to a further volume (which never appeared). This seminar covers some of the material of this second volume. Instead of following the original [1], we will read notes by Alex Wilkie (which you will receive by email) containing the proof of the o-minimality of \mathbb{R}_{an} , and then follow [4] to give a full proof the o-minimality of $(\mathbb{R}_{\text{an}}, \text{exp})$. If we have enough speakers, we finish with a proof of the o-minimal growth dichotomy and a proof of the o-minimality of $\mathbb{R}_{\text{Pfaff}}$.

In addition to giving a talk, each participant is required to submit a 4 or more page summary of their topic prepared using latex.

Prerequisites. Participants should have taken V3A5/F4A1 Mathematical Logic or a comparable course on basic model theory. A background in algebra is desirable, but not strictly necessary. This seminar complements the course V4A7 Advanced Mathematical Logic I, so preference is given to students who take is course (or have taken a similar course).

Talks. Talks marked with a (*) will only be assigned if there are enough participants.

- (1) The algebra of convergent power series, [8, Section 3 - 8] (3 talks)
- (2) Quantifier-elimination and O-minimality of \mathbb{R}_{an} , [8, Section 9] (2 talks)
- (3) An axiomatization for T_{an} , the theory of \mathbb{R}_{an} , [4, Section 2] (1 talk)
- (4) Valuation theoretic properties of models of T_{an} , [4, Section 3] (1 talk)
- (5) The theory of $(\mathbb{R}_{\text{an}}, \text{exp})$, [4, Section 4] (1 talk)
- (6) O-minimality and Hardy fields, [4, Section 5] (1 talk)
- (7) (*) Growth dichotomy for o-minimal structures [2, Section 3 after proof of 3.1, and 4.1 and 4.2] (1 talk)
- (8) (*) O-minimality of $\mathbb{R}_{\text{Pfaff}}$, [7] (3-4 talks)

REFERENCES

1. J. Denef and L. van den Dries, *p-adic and real subanalytic sets*, Ann. of Math. (2) **128** (1988), no. 1, 79–138. MR 951508
2. Chris Miller, *Basics of o-minimality and Hardy fields*, Lecture notes on o-minimal structures and real analytic geometry, Fields Inst. Commun., vol. 62, Springer, New York, 2012, pp. 43–69. MR 2976990
3. Lou van den Dries, *Tame topology and o-minimal structures*, London Mathematical Society Lecture Note Series, vol. 248, Cambridge University Press, Cambridge, 1998. MR 1633348
4. Lou van den Dries, Angus Macintyre, and David Marker, *The elementary theory of restricted analytic fields with exponentiation*, Ann. of Math. (2) **140** (1994), no. 1, 183–205. MR 1289495
5. Lou van den Dries and Chris Miller, *On the real exponential field with restricted analytic functions*, Israel J. Math. **85** (1994), no. 1-3, 19–56. MR 1264338
6. A. J. Wilkie, *Model completeness results for expansions of the ordered field of real numbers by restricted Pfaffian functions and the exponential function*, J. Amer. Math. Soc. **9** (1996), no. 4, 1051–1094. MR 1398816
7. ———, *A theorem of the complement and some new o-minimal structures*, Selecta Math. (N.S.) **5** (1999), no. 4, 397–421. MR 1740677
8. ———, *Lectures on elimination theory for semialgebraic and subanalytic sets*, 2013.