About Epicycloids and Hypocycloids *

See also the ATOs for Spherical Cycloids

Definition and tangent construction

Epicycloids resp. Hypocycloids are obtained if one circle of radius r rolls on the outside resp. inside of another circle of radius R.

In 3D-XplorMath: $r = hh,\ R = aa$.

The angular velocity of the rolling circle is fr times the angular velocity of the fixed circle (negative for hypocycloids). fr has to be an integer for the hypocycloid to be closed. The formulas do not actually roll one circle around another, they represent the curve as superposition of two rotations:

$$fr := (R - r)/(-r);$$
$$c.x := (R - r)\cos(t) + r\cos(fr \cdot t);$$
$$c.y := (R - r)\sin(t) + r\sin(fr \cdot t);$$

Double generation: If one changes the radius of the

* This file is from the 3D-XplorMath project. Please see: http://3D-XplorMath.org/

rolling circle from r to $R - r$ then these formulas are preserved, except for the parametrization speed. To view this in 3DXM replace hh by $aa - hh$.

Epicycloids are obtained if one circle of radius $r = -hh$ rolls on the outside of another circle of radius $R = aa$. The angular velocity of the rolling circle is $fr > 0$ times the angular velocity of the fixed circle (again an integer for closed epicycloids).

$$
\begin{aligned}
&fr := (R + r)/r;\\
&c.x := (R + r)\cos(t) - r\cos(fr * t);\\
&c.y := (R + r)\sin(t) - r\sin(fr * t);
\end{aligned}
$$

These formulas agree with those of the hypocycloids except for the sign of r. We view them in 3DXM by using negative hh.

We can also use a drawing stick of length $ii * r$. The default morph shows this: $0.5 < ii < 1.5$.

These more general ($ii <> 1$) rolling curves were important for Greek astronomy because the planets orbit the sun (almost) on circles. Therefore, when one looks at other planets from earth, their orbits are (almost) such rolling curves. It is no surprise that many of these curves have individual names: Astroid, Cardioid, Limacon, Nephroid are examples in 3DXM.

Tangent construction.
Rolling curves have a very simple tangent construction. The point of the rolling circle which is in contact with the base curve has velocity zero – just watch cars going by. This means that the connecting segment from this point of contact of the wheel to the endpoint of the drawing stick is the radius of the momentary rotation. The tangent of the curve which is drawn by the drawing stick is therefore orthogonal to this momentary radius.
The 3DXM-demo draws the rolling curve and shows its tangents.

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