Minisymposium 4

Spectral Theory and Ergodic Operators

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Montag, 18. September

HS XII, Hauptgebäude, Regina-Pacis-Weg

15:00 – 15:50 **Peter Stollmann** *(Chemnitz)*
Between order and disorder: Hamiltonians for quasicrystals

16:00 – 16:50 **Michael Baake** *(Bielefeld)*
Combinatorial and spectral properties of pinwheel patterns

17:00 – 17:50 **Nicolas Destainville** *(Toulouse)*
Arctic phenomena in random tilings with fixed boundaries, in dimensions 2 and 3

Dienstag, 19. September

Hörsaal 311 AVZ I, Endenicher Allee 11-13

14:30 – 14:50 **Johannes Brasche** *(Clausthal)*
Interactions along Brownian paths

15:00 – 15:50 **François Germinet** *(Cergy-Pontoise)*
Recent results on localization for random Schroedinger operators

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16:00 – 16:50 **Wolfgang König** *(Leipzig)*
The Universality Classes in the Parabolic Anderson Model

17:00 – 17:50 **Marcel Griesemer** *(Stuttgart)*
Spectral analysis of non-relativistic QED

Mittwoch, 20. September

Hörsaal 311 AVZ I, Endenicher Allee 11-13

15:00 – 15:50 **Thomas Kriecherbauer** *(Bochum)*
On the universality of random matrices

16:00 – 16:20 **Tobias Mühlenbruch** *(Clausthal)*
Hurwitz continued fractions and Ruelle's transfer operator

16:30 – 16:50 **Luka Grubisic** *(Aachen)*
Estimates for the spectral asymptotic in a Large Coupling Limit

17:00 – 17:50 **Stefan Teufel** *(Tübingen)*
Effective quantum dynamics in perturbed periodic media

Vortragsauszüge

Peter Stollmann *(Chemnitz)*
Between order and disorder: Hamiltonians for quasicrystals

The talk introduces discrete and continuum Hamiltonians for aperiodically ordered solids. As is typical in an ergodic setting, these operators come in a whole family, indexed by an apropriate dynamical system. We present some fundamental ergodic and some simple spectral properties.

Michael Baake *(Bielefeld)*
Combinatorial and spectral properties of pinwheel patterns

The classic pinwheel tiling of the plane, which is due to Conway and Radin, is defined via a primitive substitution rule based on one triangle. It contains congruent copies of this triangle in infinitely many orientations, wherefore the hull has continuous rotation symmetry. Beyond some general results on compactness, unique ergodicity and minimality, not much is known about this still somewhat enigmatic tiling.

In this talk, based on joint work with U. Grimm and D. Frettlöh, an alternative substitution rule is introduced that permits the derivation of several hitherto unknown properties and results on the combinatorics and diffraction of this tiling, together with some open conjectures.

Nicolas Destainville *(Toulouse)*

Arctic phenomena in random tilings with fixed boundaries, in dimensions 2 and 3

The effects of boundaries on macroscopic quantities such as entropy and tile statistics are discussed in random tilings of rhombi, dominoes or rhombohedra. The states that dominate the statistical ensemble of tilings are characterized. We show that under specific boundary conditions, they can display a strong structural inhomogeneity: The tilings are frozenön macroscopic regions near the boundary and only display a random character inside the ärcticfrontier. This effect is, in particular, responsible for a large difference of entropy between fixed boundary tilings and free or periodic boundary ones. We present the variational principle accounting for this arctic phenomenon. The results are demonstrated by a combination of exact and/or numerical approaches.

Johannes Brasche *(Clausthal)*

Interactions along Brownian paths

(Abstrakt lag bei Redaktionsschluss noch nicht vor.)

François Germinet *(Cergy-Pontoise)*

Recent results on localization for random Schroedinger operators

We shall review recent developments in the theory of localization for random Schroedinger operators. This includes Anderson potential as well as Poisson potentials. We shall comment on both Anderson localization and dynamical localization.

Wolfgang König *(Leipzig)*

The Universality Classes in the Parabolic Anderson Model

I shall discuss the long time behaviour of the parabolic Anderson model, the Cauchy problem for the heat equation with random potential on $\mathbb{Z}^d$. We consider general i.i.d. potentials and show that exactly *four* qualitatively different types of intermittent behaviour can occur. These four universality classes depend on the upper tail of the potential distribution: (1) tails at ∞ that are thicker than the double-exponential tails, (2) double-exponential tails at ∞ studied by Gärtner and Molchanov, (3) a new class called *almost bounded potentials*, and (4) potentials bounded from above studied by Biskup and König. The new class (3), which contains both unbounded and bounded potentials, is studied in both the annealed and the quenched setting. We show that intermittency occurs on unboundedly increasing islands whose diameter is slowly varying in time. The characteristic variational formulas describing the optimal profiles of the potential and of the solution are solved explicitly by parabolas, respectively, Gaussian densities. I shall also give a heuristic explanation in terms of the bottom of the spectrum of the Anderson Hamiltonian.

This is joint work with Remco van der Hofstad and Peter Mörters.

Marcel Griesemer *(Stuttgart)*

Spectral analysis of non-relativistic QED

The energy spectrum of non-relativistic matter coupled to quantized radiation is known to be absolutely continuous in a large interval between the ground state energy and the ionization threshold. The nature of the spectrum near the ground state energy is not known so far. In this talk I will outline two proofs for its absolute continuity. The first one is based on a new Mourre estimate and the conjuagte operator theory of Amrein et al. in the generalized version of Sahbani. The second one makes use of the BFS renormalization transformation in a novel way.

This is joint work with Juerg Froehlich and I.M. Sigal.

Thomas Kriecherbauer *(Bochum)*
On the universality of random matrices

Eigenvalues of random matrices display universal behavior in a twofold way. On the one hand local eigenvalue statistics such as the spacing distributions seem to depend for large matrix dimensions only on the symmetries of the matrices but not on the details of the chosen probability measure. On the other hand these distributions appear in many different areas of mathematics (statistics, combinatorics, number theory) and physics. In this talk both aspects of this universal behavior will be discussed.

Tobias Mühlenbruch *(Clausthal)*
Hurwitz continued fractions and Ruelle's transfer operator

Joint work with Dieter Mayer and Fredrik Strömberg (TU Clausthal)

We report a recent development concerning the transfer operator associated to a dynamical system.
We present the well known Hurwitz continued fractions and the associated dynamical system. We present also a Ruelle transfer operator L_β for this dynamical system. The transfer operator L_β is related to the Selberg ζ-function associated to the geodesic flow on the modular surface $\mathrm{SL}_2(\mathbf{Z})\backslash\mathbf{H}$. Moreover, certein eigenfunctions of the transfer operator L_β have a cocycle interpretation. These cocycles are associated to Maass cusp forms using a theorem due to Bruggeman, Lewis and Zagier. Interestingly, all these connections between the stated areas in dynamical systems, ergodic theory and number theory seem also to hold for Hecke triangle groups.
Finally, we present numerical calculations of the spectrum of the transfer operator for some selected Hecke triangle groups, pointing out the relation to Maass cusp forms.

Luka Grubisic *(Aachen)*
Estimates for the spectral asymptotic in a Large Coupling Limit

We present asymptotically sharp estimates for the convergence of eigenvalues and spectral families in the Large Coupling Limit. We reformulate the spectral convergence problem for a class of stiff problems (or non-inhibited problems in the terminology of

Sanchez-Palencia) as a task of estimating the accuracy of the Rayleigh–Ritz approximations to the spectrum of an elliptic (positive definite) operator. Our argumentation is based on recent approximation estimates, by the author, which are stable in such "ill-behaved situations". The theory is first presented in an abstract setting, since we consider applications to problems both in Quantum and Classical Mechanics. After giving the general results we present a study of the spectral asymptotics of Schroedinger operators with deep well potentials and a study of the spectral asymptotics for certain 1D approximations in the Theory of Elasticity as the diameter of the thin elastic body diminishes. Our theory is also applicable to standard singularly perturbed problems but such models will not be further considered in this talk.

Stefan Teufel *(Tübingen)*

Effective quantum dynamics in perturbed periodic media

I review results obtained with G. Panati, C. Sparber and H. Spohn concerning the effective dynamics of a single quantum particle in a slowly perturbed periodic potential. They lead to corrections to the effective Hamiltonian obtained from Peierls' substitution and to the so called "semiclassical model of solids". These corrections have a geometrical origin and add to a quantitative understanding of phenomena like Piezoelectricity or the integer quantum Hall effect.