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Optimal transportation maps for Monge-Kantorovich problem on loop groups

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Monge-Kantorovich problem is to consider how to move one distribution to another one as efficiently as possible. The efficiency is measured w.r.t. a cost function $c(x, y)$. It is naturally connected with the Wasserstein distance between two measures and also with the transportation cost inequality. In this work, we consider the Monge-Kantorovich problem on loop groups. Let G be a compact Lie group, and consider the loop group $\mathcal{L}_eG := \{\ell \in C([0, 1], G);\ \ell(0) = \ell(1) = e\}$. Let ν be the heat kernel measure at time 1. For any density function F w.r.t. ν on $\mathcal{L}_eG$ with $\mathrm{Ent}_\nu(F) < \infty$, we shall show that there exists a unique optimal transportation map $\mathcal{T} : \mathcal{L}_eG \to \mathcal{L}_eG$ which pushes ν forward to $F\nu$. Our work is based partly on McCann's result on the Riemannian manifold (2001) and partly on the Feyel and Üstünel's work (2002), where they treated the Monge-Kantorovich problem in the abstract Wiener space.