



Minisymposium 21 - Automorphic forms and their applications

Freeman Dyson's "Challenge for the Future": The mock theta functions

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In his last letter to Hardy, Ramanujan defined 17 peculiar functions which are now referred to as his mock theta functions. Although these mysterious functions have been investigated by many mathematicians over the years, many of their most basic properties remain unknown. This inspired Freeman Dyson to proclaim

"The mock theta-functions give us tantalizing hints of a grand synthesis still to be discovered. Somehow it should be possible to build them into a coherent group-theoretical structure, analogous to the structure of modular forms which Hecke built around the old theta-functions of Jacobi. This remains a challenge for the future."

Freeman Dyson, 1987, Ramanujan Centenary Conference

Here we announce a solution to Dyson's "challenge for the future" by providing the "coherent group-theoretical structure" that Dyson desired in his plenary address at the 1987 Ramanujan Centenary Conference. In joint work with Ken Ono, we show that Ramanujan's mock theta functions, as well a natural generalized infinite class of mock theta functions may be completed to obtain Maass forms, a special class of modular forms. We then use these results to prove theorems about Dyson's partition ranks. In particular, we shall prove the 1966 Andrews-Dragonette Conjecture, whose history dates to Ramanujan's last letter to Hardy, and we shall also prove that Dyson's ranks 'explain' Ramanujan's partition congruences in an unexpected way.